Math 1, Lecture 3
Precalculus

Sample Midterm 2

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books, notes, or calculators. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: Solutions

UID: _____

Section: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

A javelin is thrown into the air. Its height is given by $h(x) = -\frac{1}{10}x^2 + x + 20$, where x is the horizontal distance in feet from the point at which the javelin was thrown.

(a) [2pts.] How high is the javelin when it is thrown?

(b) [4pts.] What is the maximum height the javelin reaches?

(c) [4pts.] How far from the thrower does the javelin strike the ground?

(a) $h(0) = 20$ ft

(b) Maximum occurs at vertex.

$$x = \frac{-b}{2a} = \frac{-1}{\frac{-2}{10}} = \frac{10}{2} = 5 \text{ feet away horizontally}$$

$$h(5) = \frac{-1}{10}(25) + 5 + 20$$

$$= \frac{-5}{2} + 5 + 20$$

$$= \boxed{22.5 \text{ Ft}}$$

(c) Want $0 = \frac{-1}{10}x^2 + x + 20$

$$0 = \frac{-1}{10}(x^2 - 10x - 200)$$

$$0 = \frac{-1}{10}(x - 20)(x + 10)$$

$\boxed{x = 20}$ ~~$x = -10$~~ Not reasonable

The javelin strikes the ground 20 feet away.

Problem 2.

Let $f(x)$ be a quintic polynomial with roots of multiplicity 1 at $x = -5$, $x = 3$, and $x = 2$, a root of multiplicity 2 at $x = -1$, and y-intercept 15.

(a) [3pts.] Give an equation for $f(x)$.

(b) [3pts.] Describe, using the notation introduced in class, the long-term behavior of $f(x)$.

(c) [4pts.] Draw a graph of $f(x)$.

(a) $F(x) = a(x+5)(x-3)(x-2)(x+1)^2$

$15 = F(0) = a(5)(-3)(-2)(1)$

$15 = 30a$

$\frac{1}{2} = a$

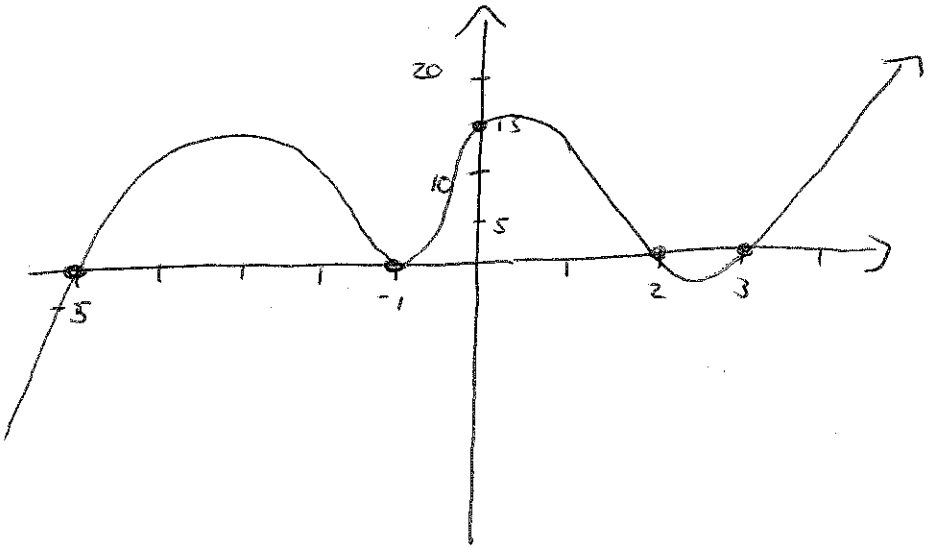
$$F(x) = \frac{1}{2}(x+5)(x-3)(x-2)(x+1)^2$$

(b) Leading term is $\frac{1}{2}x^5$

As $x \to \infty$, $F(x) \to \infty$.

As $x \to -\infty$, $F(x) \to -\infty$.

(c)



Problem 3.

Consider the rational function

$$g(x) = \frac{x^3 - 2x^2 + x}{9 - x^2}$$

(a) [5pts.] Find all asymptotes and intercepts of $g(x)$.

(b) [5pts.] Solve the inequality $g(x) \leq 0$.

(a) $g(x) = \dfrac{x(x^2-2x+1)}{(3+x)(3-x)} = \dfrac{x(x-1)^2}{-(x+3)(x-3)}$

Vertical intercept $y = g(0) = 0$

Vertical asymptotes $x = 3,\ x = -3$

Horizontal intercepts $x = 0,\ x = 1$

No horizontal asymptote (Degree of numerator > degree of denominator)

(b)

	$(-\infty, -3)$	$(-3, 0)$	$(0, 1)$	$(1, 3)$	$(3, \infty)$
x	$-$	$-$	$+$	$+$	$+$
$(x-1)^2$	$+$	$+$	$+$	$+$	$+$
-1	$-$	$-$	$-$	$-$	$-$
$(x+3)$	$-$	$+$	$+$	$+$	$+$
$(x-3)$	$-$	$-$	$-$	$-$	$+$
Total	$+$	$-$	$+$	$+$	$-$

$(-3, 0] \cup \{1\} \cup (3, \infty)$ } Intervals where g is negative, plus horizontal intercepts.

Problem 4.

The value of a mint edition comic book is ten dollars in 2000 and has appreciated to thirty dollars by 2012.

(a) [4pts.] Find an exponential model for the value of the comic book.

(b) [3pts.] What do you expect the value is today?

(c) [3pts.] At what time does your model predict the comic book will be worth fifty dollars? Leave your answer in unsimplified form.

(a) Let $t=0$ be 2010,

$(0, 10)$

$(2, 30)$

$V(t) = 10(b^t)$

$30 = V(2) = 10b^2$

$3 = b^2$

$\sqrt{3} = b$

$V(t) = 10(\sqrt{3}^t)$

(b) In 2014, $t=4$. $V(4) = 10(\sqrt{3})^4 = 10(9) = \$90.$

(c) $50 = 10(\sqrt{3})^t$

$5 = (\sqrt{3})^t$

$\ln 5 = \ln(\sqrt{3}^t)$

$\ln 5 = \frac{t}{2} \ln 3$

$\frac{2 \ln 5}{\ln 3} = t$ [Also Fine: $t = 2\log_3 5$ or $t = \frac{2\log 5}{\log 3}$]

Problem 5.

For each equation, either solve for x, or explain why this is impossible.

(a) [5pts.] $\log_6(x-2)+\log_6(x+3)=1$

(b) [5pts.] $3e^x+5=2$

(a) $\log_6(x-2)+\log_6(x+3)=1$

$\log_6((x-2)(x+3))=1$

$(x-2)(x+3)=6^1$

$x^2+x-6=6$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ $\boxed{x=3}$ ← Single solution

$\times$

Not reasonable, $\ln(-4-2)=\ln(-6)$ does not exist.

(b) $3e^x+5=2$

$3e^x=-3$

$e^x=-1$

$\times$

Not possible, $e^x>0$ For all x.